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Aspects of Turbulent Transport

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Turbulence is an ubiquitous phenomenon in plasmas, which have usually large amounts of free energy available that can drive the turbulent velocity fluctuations. Universal properties of turbulence are its abilities to mix and to transport. The turbulent transport is often found not to be described by a transport coefficient, as Fick's law suggests. We discuss the consequences of this and the relationship between passive particle and plasma transport, that is mixing and transport.

1. Introduction

A characteristic feature of turbulence is the ability to disperse and mix particles and heat. This process is rather complicated even under idealized conditions of homogeneous and isotropic turbulence and is after many years of research still far from being understood in general. Covering a variety of important areas from the diffusion of pollutants in environmental flows to the particle and heat diffusion in magnetized plasmas this topic connects basic research with applications.

For magnetized plasmas it is agreed that the enhanced levels of cross-field transport (anomalous transport), observed in a wide variety of devices including tokamaks and stellarators is due to low-frequency, electrostatic, micro-turbulence, with the $E \times B$-drift as the dominating velocity. A good candidate to understand and explain the anomalous transport from first principles is drift-wave turbulence [1, 2, 3]. We investigate turbulence of magnetized plasmas and discuss the influence of structures on the transport.

2. Particle Dispersion...

The mean square displacement of an ensemble of particles in a velocity field $\vec{u}(\vec{x},t)$ is given by [5]:

$$\langle \vec{x}^2(t) \rangle = 2t \langle u^2 \rangle \int_0^t \left(1 - \frac{\tau}{t}\right) R_L(\tau) d\tau \qquad (1)$$

with $\langle \vec{u}(\vec{x}(t)) \cdot \vec{u}(\vec{x}(t'),t') \rangle = R_L(t-t') \langle u^2 \rangle$. Here R_L is the normalized Lagrangian velocity correlation. For very short times the correlation is close to unity in the so-called ballistic limit:

$$\langle \vec{x}^2(t) \rangle = t^2 \langle u^2 \rangle \ . \qquad (2)$$

For very long times the integral evaluates to the Lagrangian integral τ_l

$$\int_0^t R_L(\tau) d\tau \approx \int_0^\infty R_L(\tau) d\tau =: \tau_L \ . \qquad (3)$$

and provided that τ_l is finite we obtain

$$\langle \vec{x}^2(t) \rangle = 2t\tau_L \langle u^2 \rangle = 2\tau_L D \qquad (4)$$

This limit ($t \gg \tau_L$) is often called the diffusion limit. Note that τ_L differs from the Eulerian correlation time available to laboratory measurements. For homogeneous, isotropic turbulence and times longer than the Lagrangian correlation time the particle diffusion will thus be normal. However, there might be large intermediate ranges depending on the correlation times and scales, especially if the correlations reach the spatial or temporal extend of the system.

3. ... and Transport

The question if test particle transport is connected to the turbulent radial transport of plasma

$$\Gamma = \langle n v_r \rangle$$

by the $E \times B$ velocity ($v_r = -\partial_y \phi$) is far from trivial. We consider the Hasegawa-Wakatani equations (HWE) [4] as minimal model for drift-wave type plasma turbulence:

$$\partial_t n + \partial_y \phi + \{\phi, n\} = -C\,(n - \phi) + \mu \nabla^2 n \ , \qquad (5)$$

$$\partial_t \nabla^2 \phi + \{\phi, \nabla^2 \phi\} = -C\,(n - \phi) + \mu \nabla^4 \phi \qquad (6)$$

The deviation from adiabaticity, given by the parameter $1/C$, leads to an instability. The HWE Eqs. (5, 6) contain two limits: i) $C \to 0$, where the density and potential decouples, thus the system describes standard 2D Navier-Stokes turbulence with n passively advected. ii) $C \to \infty$, which corresponds to the adiabatic density response $\phi \approx n$ and the HWE reduce to the Hasegawa-Mima equation (HME).

The evolution of the energy $E \equiv \frac{1}{2} \int ((\nabla \phi)^2 + n^2) dV$ and generalized enstrophy $W \equiv \frac{1}{2} \int (\omega - n)^2 dV$ is governed by

$$d_t E = \int -n \partial_y \varphi - C\,(n - \phi)^2 - \mu(\omega^2 + (\nabla n)^2) d^2 r \ , \qquad (7)$$

$$d_t W = \int -n \partial_y \phi - \mu (\nabla (n - \omega))^2 d^2 r \ . \qquad (8)$$

Here $\omega = \nabla \times \mathbf{v} = \nabla^2 \varphi$ is the vorticity. The driving term is the integrated $E \times B$-flux

$$\Gamma_0 = \int \Gamma d^2 r = \int n u \varphi d^2 r = \int -n \partial_y \varphi d^2 r,$$

which mediates the instability when $n \neq \phi$. In the inviscid limit the HWE has a Lagrangian conserved quantity $\Pi = (\omega - n + x)$. We employ the conservation of Π to estimate the radial dispersion of the fluid elements. From $x - x_0 = -(\xi - \xi_0)$, where $\xi = \omega - n$. We obtain:

$$\langle (x - x_0)^2 \rangle_p = \langle \xi^2 \rangle_p + \langle \xi_0^2 \rangle_p - 2 \langle \xi_0 \xi \rangle_p, \qquad (9)$$

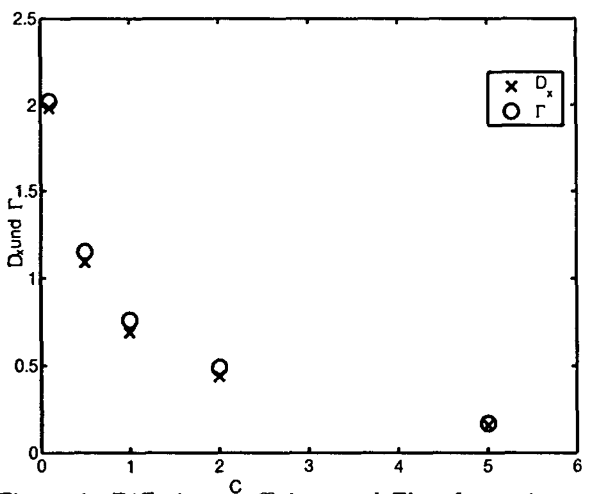

Figure 1: Diffusion coefficient and Flux for various values of C.

where the average $\langle\cdot\rangle_p$ is taken over all fluid elements/particles. If the turbulence is homogeneous, we can replace the averaging over the fluid elements with averaging over the fields ($\langle\cdot\rangle_p = \langle\cdot\rangle_f$). In the time asymptotic limit we may then assume that the last term on the rhs vanishes (this designates the correlation between the fluid PV and the initial fluid PV). Taking the time derivative and using Eq. (8) we obtain:

$$d_t\langle(x-x_0)^2\rangle_p = d_t\langle\xi^2\rangle_f = 2d_tW. \tag{10}$$

Finally we arrive at

$$D_x = \Gamma_0\,, \tag{11}$$

what is confirmed by numerical experiment see Figure 1.

4. Transport PDF

In models where the fluctuations are driven by a local instability and the density fluctuation-level is small compared to the background density the probability distribution function (PDF) of as well density and radial velocity is given by a normal distribution.

When evaluating for the PDF of Γ we note that the folding of two normal distributions is not a normal distribution. Moreover if n and ϕ are correlated the average of the PDF may become non-zero and lead to a net transport.

If the transport PDF is evaluated not for the local values of the transport, but averaged over some domain, the transport PDF has to converge to a stable distribution. If the second order moment is finite, the central limit theorem tells us that this is a normal distribution. If the second order moment does not exist, than we have a Levy-Pareto stable PDF and thus expect a power law tail in the PDF, as long as all the averaging procedures are carried out for times/lengths well below the correlation length/time-scales. A power law tail in the PDF of the averaged flux reflects thus the existence of long range correlations in the system, respectively that the turbulence is intermittent. Correlations that can manifest themselves via the occurrence of coherent structures within the turbulence.

5. Turbulence and Structures: The quest for correlations

Two dimensional turbulence, as the turbulence of magnetized plasmas, is characterized by a dual cascade: of enstrophy to the small scales and energy to large scales. Moreover if the vorticity is a function of the potential, then the convective non-linearity vanishes and thus turbulent de-correlation mechanisms are no longer very effective. The regions of reduced nonlinearity and correlated potential and vorticity are also known as coherent structures. These structures form out of the turbulence and can survive for long times compared to the turbulence de-correlation time. Tracking particles in the turbulence and stating if they are trapped inside structures we could find that this introduces an intermediate regime for the transport [6].

6. Turbulent Equipartition

Usually turbulence takes place in an confined vessel with boundary conditions and the plasma is continuously driven. The arguments used above are then no longer valid. Indeed based on the conservation of Lagrangian invariants we could show, that the turbulence sets up characteristic gradients, independently of the magnitude of the flux generated. Clearly this breaks Fick's law. Moreover it leads to a bursty type of transport, where plasma is pushed outwards in large structures [7].

7. Summary

It is found that as long as the spatial/temporal extend of the plasma is close to correlation times/length scales the turbulent structures induce a power law tail in the PDF of the transport. This transport is then not described by Fick's law on diffusion and turbulent profiles might be sustained at various levels of flux through the system.

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